Portfolio Returns

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BUSI 448: Investments

Where are we?

Last time:

- Calculating returns
- Fetching data
- Summarizing returns

Today:

- Returns of portfolios
- Portfolio expected return
- Portfolio standard deviation

Returns of Portfolios

Portfolios

- Portfolio are combinations of underlying assets
- Given return properties of the underlying assets, what are the return properties of their combination?

Expected Return of Portfolio of N Assets

$$E[r_p] = \sum_{i=1}^{N} w_i E[r_i]$$

- w_i is the portfolio weight of asset i
- $E[r_i]$ is the expected return of asset i
- The portfolio is fully invested: $\sum_i w_i = 1$
- Notation: $E(r_p) = \mu_i$

Variance of Portfolio of N Assets

$$\text{var}[r_p] = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \text{cov}[r_i, r_j]$$

- w_i is the portfolio weight of asset i
- $\text{cov}[r_i, r_j]$ is the covariance between assets i and j
- Recall that $\text{cov}[r_i, r_i] = \text{var}[r_i]$ and $\text{sd}[r_i] = \sqrt{\text{var}[r_i]}$
- Notation: $\text{var}[r_p] = \sigma_p^2$; $\text{cov}[r_i, r_j] = \sigma_{i,j}$; $\text{sd}[r_p] = \sigma_p$

Variance of Portfolio of N Assets: A Matrix View

$$\text{var}[r_p] = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \text{cov}[r_i, r_j]$$

$w_1 w_1 \text{cov}[r_1, r_1]$	$w_1 w_2 \text{cov}[r_1, r_2]$	$w_1 w_3 \text{cov}[r_1, r_3]$
$w_2 w_1 \text{cov}[r_2, r_1]$	$w_2 w_2 \text{cov}[r_2, r_2]$	$w_2 w_3 \text{cov}[r_2, r_3]$
$w_3 w_1 \text{cov}[r_3, r_1]$	$w_3 w_2 \text{cov}[r_3, r_2]$	$w_3 w_3 \text{cov}[r_3, r_3]$

Variance of Portfolio of N Assets: A Matrix View

$$\text{var}[r_p] = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \text{cov}[r_i, r_j]$$

$w_1^2 \text{var}[r_1]$	$w_1 w_2 \text{cov}[r_1, r_2]$	$w_1 w_3 \text{cov}[r_1, r_3]$
$w_2 w_1 \text{cov}[r_2, r_1]$	$w_2^2 \text{var}[r_2]$	$w_2 w_3 \text{cov}[r_2, r_3]$
$w_3 w_1 \text{cov}[r_3, r_1]$	$w_3 w_2 \text{cov}[r_3, r_2]$	$w_3^2 \text{var}[r_3]$

Variance of Portfolio of N Assets: A Matrix View

$$\text{var}[r_p] = \sum_{i=1}^{N} w_i^2 \text{var}[r_i] + 2 \sum_{j>i} w_i w_j \text{cov}[r_i, r_j]$$

$w_1^2 \text{var}[r_1]$	$w_1 w_2 \text{cov}[r_1, r_2]$	$w_1 w_3 \text{cov}[r_1, r_3]$
$w_2 w_1 \text{cov}[r_2, r_1]$	$w_2^2 \text{var}[r_2]$	$w_2 w_3 \text{cov}[r_2, r_3]$
$w_3 w_1 \text{cov}[r_3, r_1]$	$w_3 w_2 \text{cov}[r_3, r_2]$	$w_3^2 \text{var}[r_3]$

Example: Equal-weighted portfolio of two assets

- Expected Return

$$
\begin{aligned}
E[r_p] &= w_1 E[r_1] + w_2 E[r_2] \\
&= 0.5 E[r_1] + 0.5 E[r_2]
\end{aligned}
$$

- Portfolio Variance

$$
\begin{aligned}
\text{var}[r_p] &= w_1^2 \text{var}[r_1] + w_2^2 \text{var}[r_2] + 2 w_1 w_2 \text{cov}[r_1, r_2] \\
&= 0.5^2 \text{var}[r_1] + 0.5^2 \text{var}[r_2] + 2 \cdot 0.5 \cdot 0.5 \text{cov}_{12} \\
&= 0.25 \text{var}[r_1] + 0.25 \text{var}[r_2] + 0.5 \text{cov}[r_1, r_2]
\end{aligned}
$$

Variance of Portfolio of N Assets: Matrices

$$\text{var}[r_p] = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \text{cov}[r_i, r_j] = w' V w$$

- Portfolio weights vector

$$w' = [w_1 \; w_2 \ldots w_N]$$

- Covariance matrix of returns:

$$V = \begin{bmatrix} \text{var}[r_1] & \text{cov}[r_1, r_2] & \ldots & \text{cov}[r_1, r_N] \\ \text{cov}[r_2, r_1] & \text{var}[r_2] & \ldots & \text{cov}[r_2, r_N] \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}[r_N, r_1] & \text{cov}[r_N, r_2] & \ldots & \text{var}[r_N] \end{bmatrix}$$

Covariance and Correlation

- Covariance: absolute degree of co-movement between two assets
- Correlation: relative degree of co-movement between two assets

$$\text{corr}[r_i\,, r_j] = \rho_{ij} = \frac{\text{cov}[r_i\,, r_j]}{\text{sd}[r_i] \cdot \text{sd}[r_j]}$$

- What are the possible values for ρ?

Portfolio Statistics in Python

Portfolio expected return in Python

```python
import numpy as np

# Expected returns
mns = np.array([0.10, 0.05, 0.07])

# Portfolio weights
wgts = np.array([0.25, 0.5, 0.25])

#Portfolio expected return
port_expret = wgts @ mns
```

Portfolio variance: matrix approach

Given a covariance matrix V:

$$V = \begin{bmatrix} \text{var}[r_1] & \text{cov}[r_1, r_2] & \dots & \text{cov}[r_1, r_N] \\ \text{cov}[r_2, r_1] & \text{var}[r_2] & \dots & \text{cov}[r_2, r_N] \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}[r_N, r_1] & \text{cov}[r_N, r_2] & \dots & \text{var}[r_N] \end{bmatrix}$$

and a vector of portfolio weights

$$w' = [w_1 \, w_2 \dots w_N] \, ,$$

The portfolio variance is the matrix product:

$$\text{var}[r_p] = w' V w \, .$$

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Portfolio variance in Python: Inputs

```python
import numpy as np

##### Inputs
# Standard deviations
sds = np.array([0.20, 0.12, 0.15])

# Correlations
corr12 = 0.3
corr13 = 0.3
corr23 = 0.3

# Portfolio weights
wgts = np.array([0.25, 0.5, 0.25])
```

Covariance matrix: method 1

```
##### Method 1 to calculate covariance matrix
# Covariances
cov12 = corr12 * sds[0] * sds[1]
cov13 = corr13 * sds[0] * sds[2]
cov23 = corr23 * sds[1] * sds[2]
# Covariance matrix
cov = np.array([[sds[0]**2,  cov12,     cov13], \
                [cov12,      sds[1]**2, cov23], \
                [cov13,      cov23,     sds[2]**2]])
```

Covariance matrix: method 2

```
##### Method 2 to calculate covariance matrix
# Correlation matrix
C   = np.identity(3)
C[0, 1] = C[1, 0] = corr12
C[0, 2] = C[2, 0] = corr13
C[1, 2] = C[2, 1] = corr23
# Covariance matrix
cov = np.diag(sds) @ C @ np.diag(sds)
```

Portfolio risk in Python

```
##### Portfolio risk measures
# Portfolio variance
port_var = wgts @ cov @ wgts

# Portfolio standard deviation
port_sd  = np.sqrt(port_var)
```

For next time: Equity Markets