Portfolios: Sensitivity to Inputs

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BUSI 448: Investments

Where are we?

Last time:

- Simulation
- Investing over multiple periods
- Rebalancing

Today:

- Sensitivity of mean-variance optimization to inputs
- Dealing with estimation error of inputs
- Empirical and simulated performance

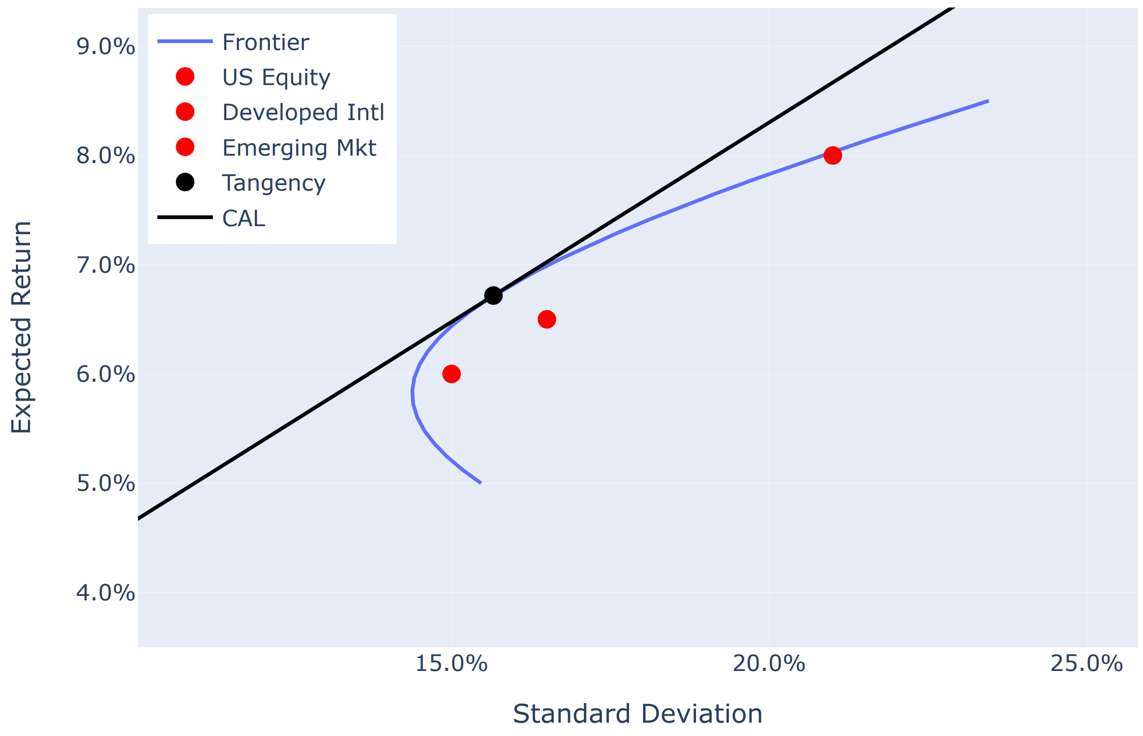
Input sensitivity

Portfolio optimization inputs

- Set of expected returns for assets
- Set of std deviations (variances) for assets
- Set of correlations (covariances) across assets

How good are we at estimating these things?

Three-asset example



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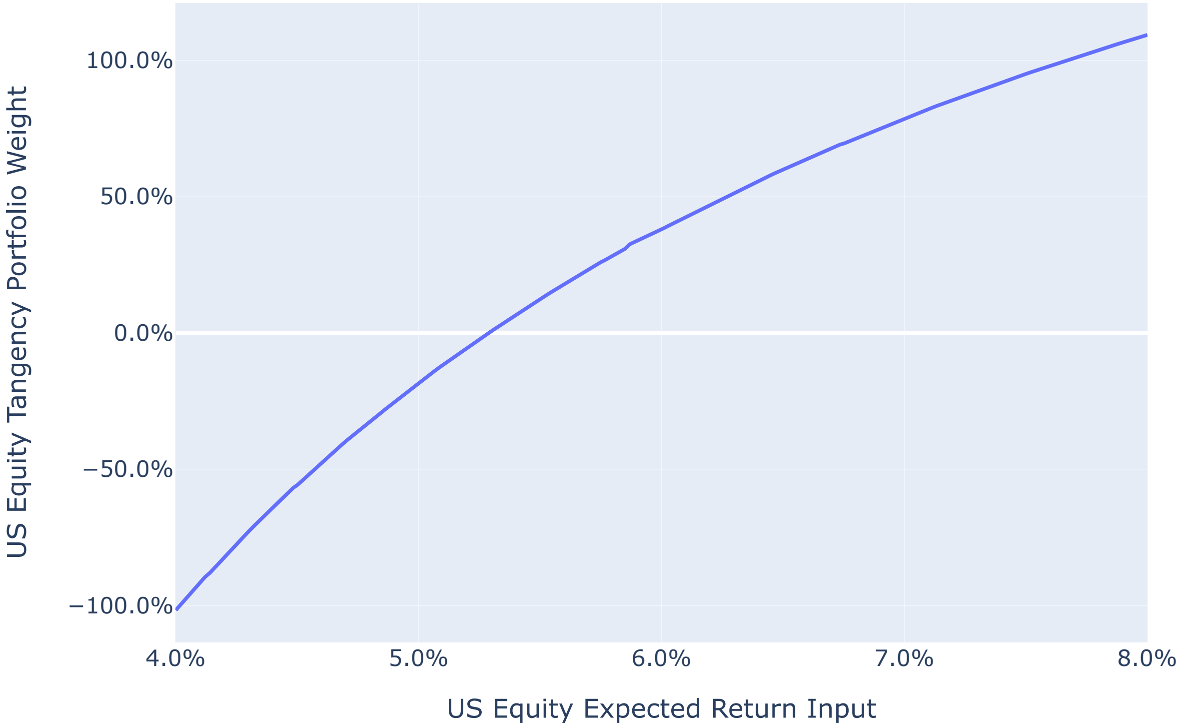
Sensitivity to expected returns



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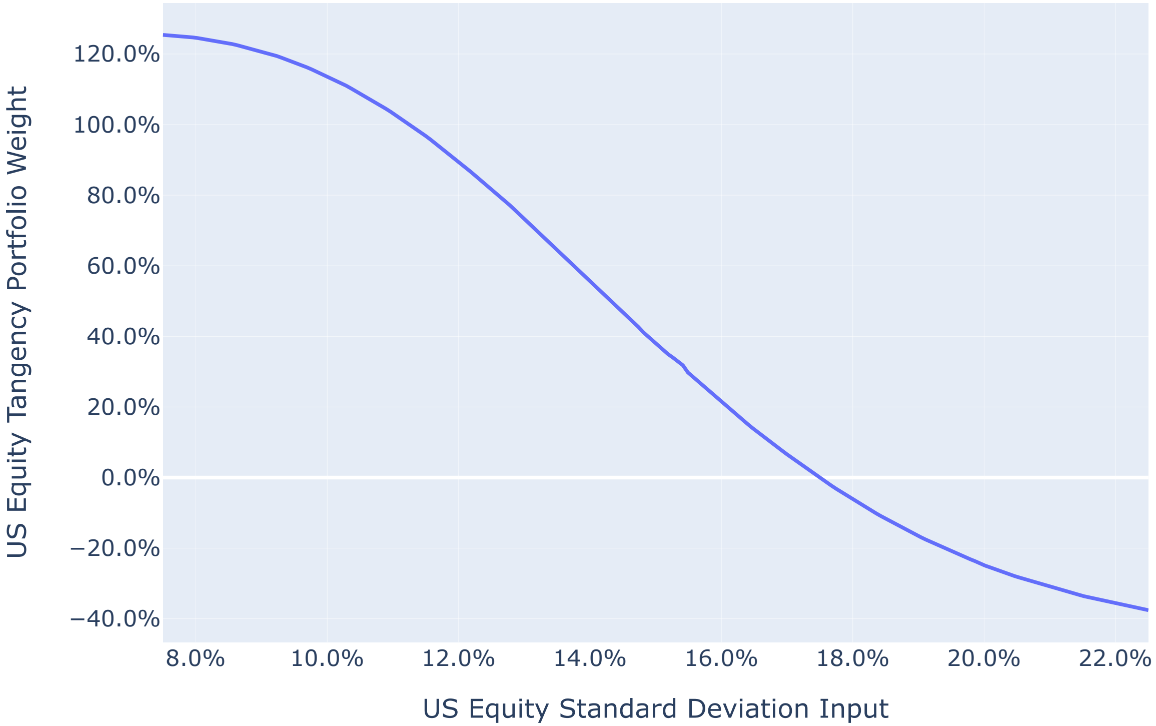
Sensitivity to standard deviations



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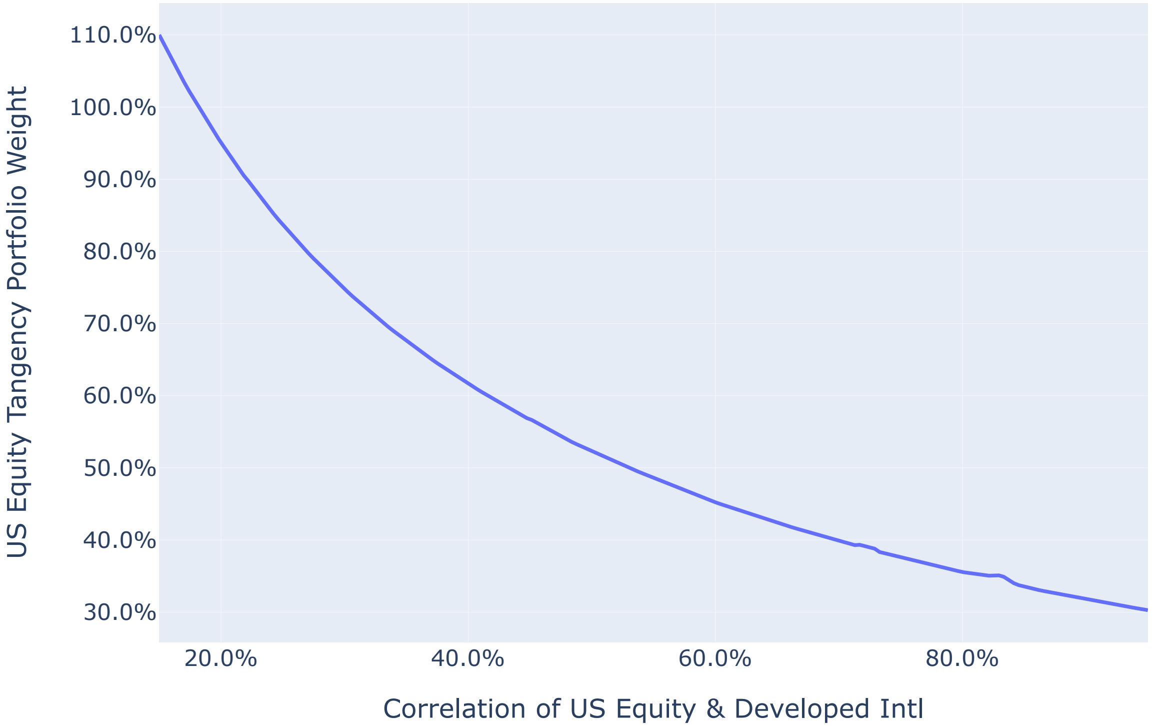
Sensitivity to correlations



The Error-Maximization Problem

Mean-variance portfolio optimization:

- Will tilt too heavily toward assets with estimated expected returns above true expected returns ($\hat{\mu} > \mu$)
- Will tilt too heavily toward assets with diversification benefits greater than true benefits ($\widehat{\text{cov}}_{ij} < \text{cov}_{ij}$)
- May try to short assets with diversification benefits lower than true benefits ($\widehat{\text{cov}}_{ij} > \text{cov}_{ij}$)

Dealing with Estimation Error

Position limits

Short-selling constraints

Prevent hedging positions due to overestimated covariances and/or underestimated $E[r]$

Maximum positions

Prevent overweighting due to overestimated $E[r]$ and/or underestimated covariances

Shrinkage

- Shrink extreme inputs toward some more moderate input
- Example: CAPM betas

$$\beta_{\text{adj}} = 0.67 \cdot \hat{\beta} + 0.33 \cdot 1$$

- There are some fairly sophisticated shrinkage techniques for the covariance matrix.

Use models to infer expected returns

Black-Litterman

Use market value weights to back out $E[r_i]$'s via CAPM

Then add alphas to expected returns

Treynor-Black

Consider benchmark index as an asset

Use expected alphas to create an active portfolio

Combine index and active portfolio optimally

Factor models

- Can be used to estimate both $E[r]$'s and correlations
- **Market Model/CAPM**:

$$E[r_i] = r_f + \beta E[r_{\text{mkt}} - r_f]$$

$$\text{cov}_{ij} = \beta_i \beta_j \sigma^2_{\text{mkt}}$$

- Can dramatically reduce the number of estimated parameters
- We will discuss (multi-)factor models beyond CAPM

Don't even try to estimate some inputs!

Global minimum variance

assume all $E[r_i]$'s equal

Risk parity

assume all $E[r_i]$'s equal and all $\rho_{ij} = 0$

Equal-weighted portfolio

assume all $E[r_i]$'s, $\text{sd}[r_i]$'s equal; all $\rho_{ij} = 0$

Empirical Performance of Historical Plug-in Estimators

Historical Plug-in Estimators

Expected return

use historical arithmetic average return

Standard deviation

use historical standard deviation

Correlations

use historical pair-wise correlation

Stocks, Bonds, and Gold

Let's run a backtest of annual optimization of portfolios of the following asset classes:

- Stocks
- Treasury bonds
- Corporate bonds
- Gold

We'll use four strategies for input estimation.

Strategy 1: Est-All

- use historical data to estimate expected returns, standard deviations, and correlations.
- optimal risky portfolio is the tangency portfolio
- scale tangency up or down depending on risk aversion or target expected return

Strategy 2: Est-SD-Corr

- use historical data to estimate standard deviations and correlations
- assume expected returns are the same across all assets.
- optimal risky portfolio is the global minimum variance portfolio.
- for the purposes of determining optimal capital allocation, use the cross-sectional average of the historical time-series average return as the expected return input.

Strategy 3: Est-SD

- use historical data to estimate standard deviations only
- assume correlations across assets are zero
- assume expected returns are the same across all assets
- for the purposes of determining optimal capital allocation, use the cross-sectional average of the historical time-series average return as the expected return input.

Strategy 4: Est-None

- do not use historical data to estimate expected returns, standard deviations, or correlations.
- the optimal portfolio is an equal-weighted portfolio of the assets ($1/N$ portfolio).
- for the purposes of determining optimal capital allocation, use the cross-sectional average of the historical time-series average return as the expected return input.

To notebook #1

Empirical Performance of Portfolio Constraints

Industry Portfolios

Let's return to our 48 industry portfolio example.

- Using full sample means, standard deviations, and correlations suggested that allowing short selling could improve mean-variance efficiency.
- Let's consider how this would have fared in an **out-of-sample** context.
- We will use expanding windows to estimate inputs.

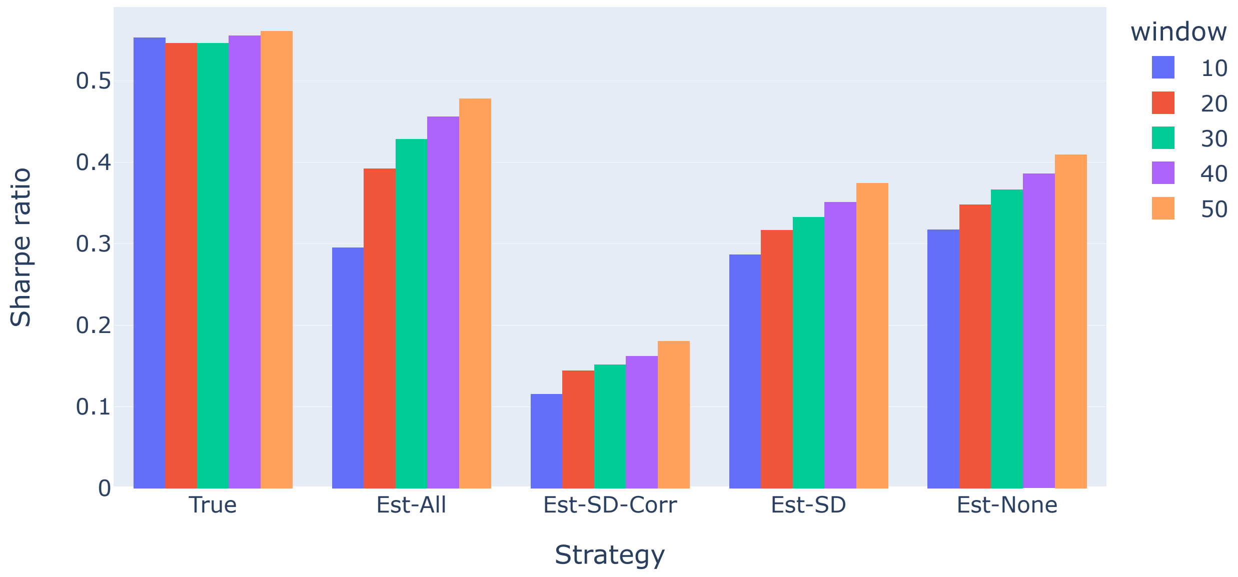
To notebook #2

Simulated Performance of Historical Plug-in Estimators

Length of Estimation Window

- Use last T years to estimate inputs (rebalance each year)
- Consider windows of 10, 20, 30, 40, and 50 years
- Scenarios with more or less dispersion in true expected returns

Higher $E[r]$ dispersion



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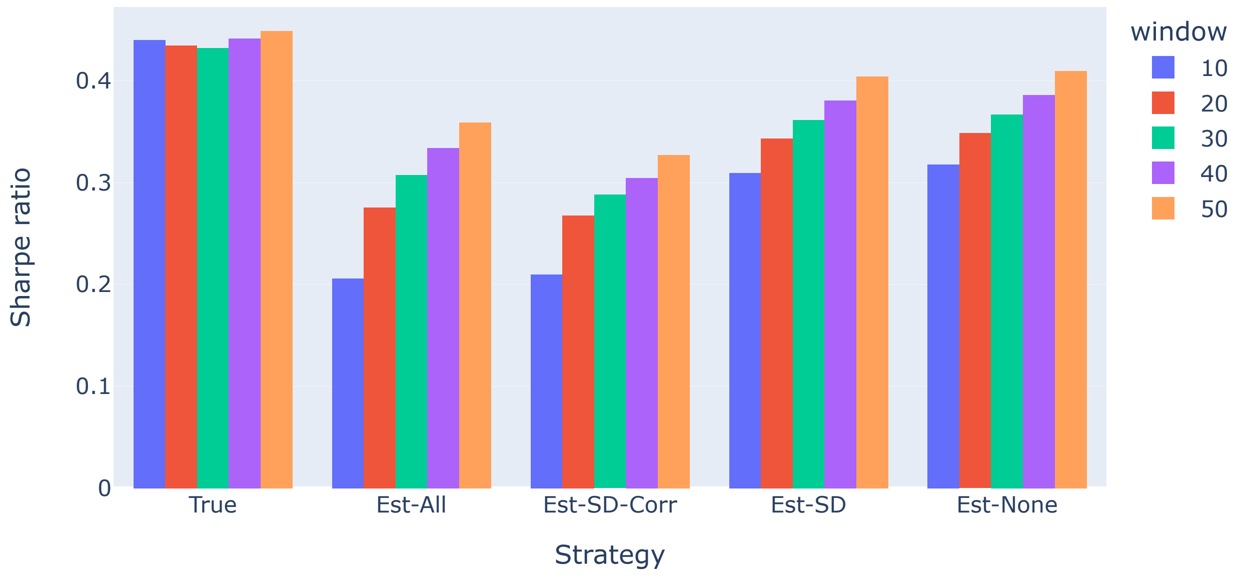
Lower $E[r]$ dispersion



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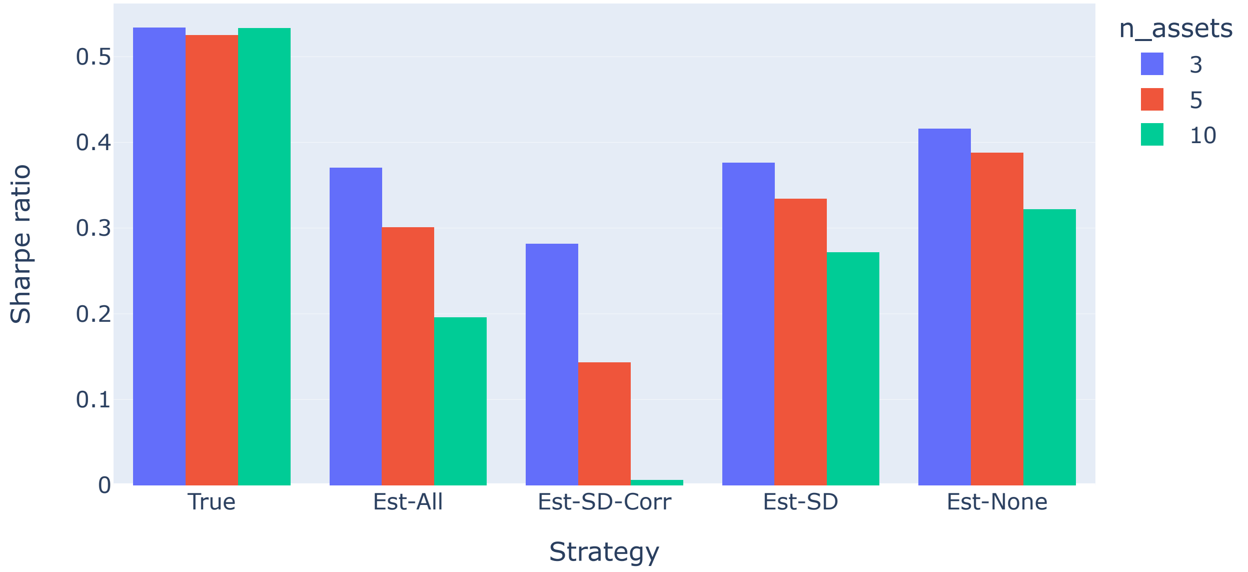
Number of Assets

- 3, 5, or 10 assets
- Estimation window of 30 years
- Investment period of 50 years
- Theoretical Sharpe ratio of tangency portfolio is the same

Number of Assets



For next time: Market Model Regression