Multifactor Models

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BUSI 448: Investments

Where are we?

Last time:

- The cross-section of expected stock returns
- Portfolio sorts
- Cross-sectional regression

Today:

- Multifactor models
- Estimating expected returns
- Characteristic-based models

Multifactor models

Expected returns

- We are interested in characterizing the risk premium for stocks

$$E[r] = r_f + \text{risk premium}$$

- Empirically, the CAPM fares poorly in this regard.
- Today, we will explore some alternatives.

Fama-French 3-factor model

Motivated by the size and value anomalies, Fama and French argued for a three **factor** model.

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t + \varepsilon_{i,t}$$

- Size factor: SMB (Small Minus Big)
- Value factor: HML (High Minus Low)
- Widely used asset-pricing model for stocks and for evaluation of asset managers

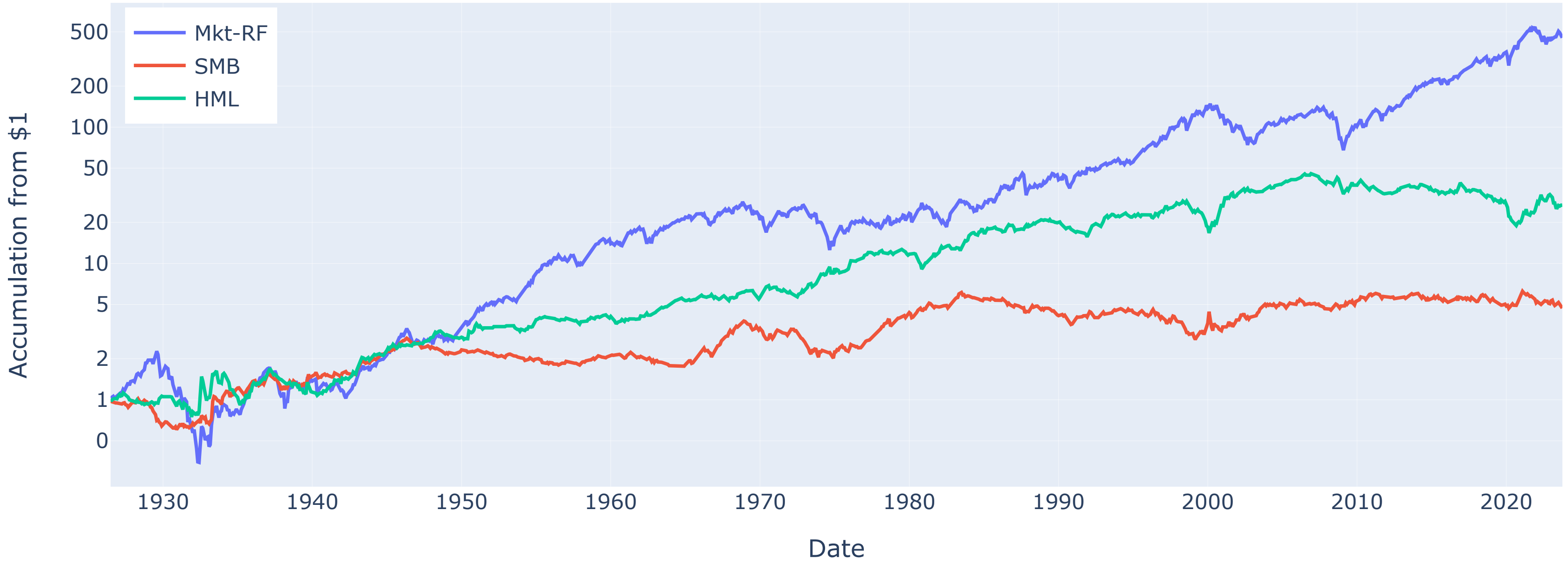
SMB and HML

Form 6 portfolios on size (mkt cap) and value (B/M ratio)

	Low B/M	Medium B/M	High B/M
Small	Small growth		Small value
Large	Large growth		Large value

- SMB: $(0.5 \cdot SG + 0.5 \cdot SV) - (0.5 \cdot LG + 0.5 \cdot LV)$
- HML: $(0.5 \cdot SV + 0.5 \cdot LV) - (0.5 \cdot SG + 0.5 \cdot LG)$

SMB + HML cumulative returns



What are the CAPM alphas of HML and SMB?

$$SMB_t = \alpha_{\text{SMB}} + \beta_{\text{SMB}}(R_{m,t} - R_{f,t}) + \varepsilon_{i,t}$$

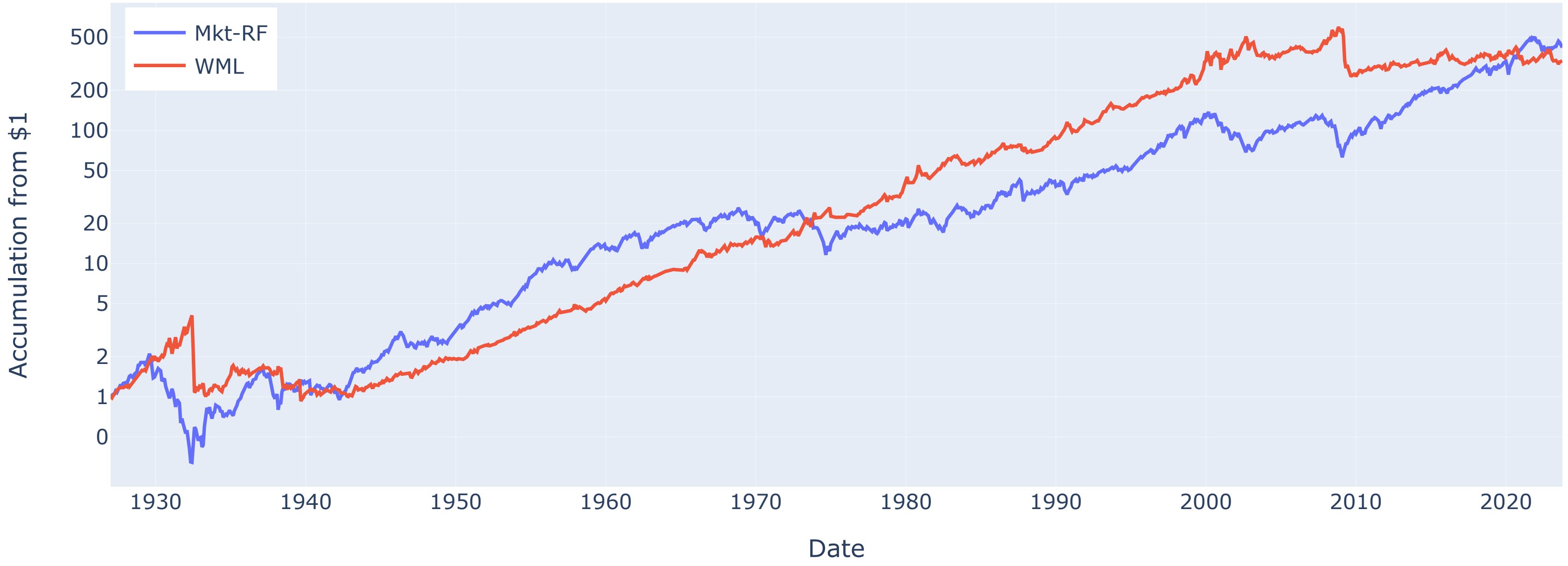
$$HML_t = \alpha_{\text{HML}} + \beta_{\text{HML}}(R_{m,t} - R_{f,t}) + \varepsilon_{i,t}$$

- Let's look at today's first notebook.
- Recall: non-zero alphas mean that the market portfolio is not mean-variance efficient
  - Investing in a portfolio of the market and a positive alpha portfolio leads to a higher Sharpe ratio.

Momentum

- Consider sorting stocks based on their returns over the past year
- Call the top performers "winners"
- Call the bottom performers "losers"
- A portfolio that goes long "winners" and short "losers" outperforms
- This is known as a **momentum** strategy

Momentum cumulative returns



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Momentum alphas

Can market risk exposure explain momentum?

$$WML_t = \alpha_{\text{WML}} + \beta_{\text{WML}}(R_{m,t} - R_{f,t}) + \varepsilon_{i,t}$$

What about the size and value factors?

$$WML_t = \alpha_{\text{WML}} + \beta_{\text{WML}}(R_{m,t} - R_{f,t}) + s_{\text{WML}} SMB_t + h_{\text{WML}} HML_t + \varepsilon_{i,t}$$

Fama-French-Carhart model

The FFC model augments the Fama-French-Carhart model with a momentum factor.

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + s_i SMB_t + h_i HML_t + m_i WML_t + \varepsilon_{i,t}$$

- Size factor: SMB (Small Minus Big)
- Value factor: HML (High Minus Low)
- Momentum factor: WML (Winners Minus Losers)

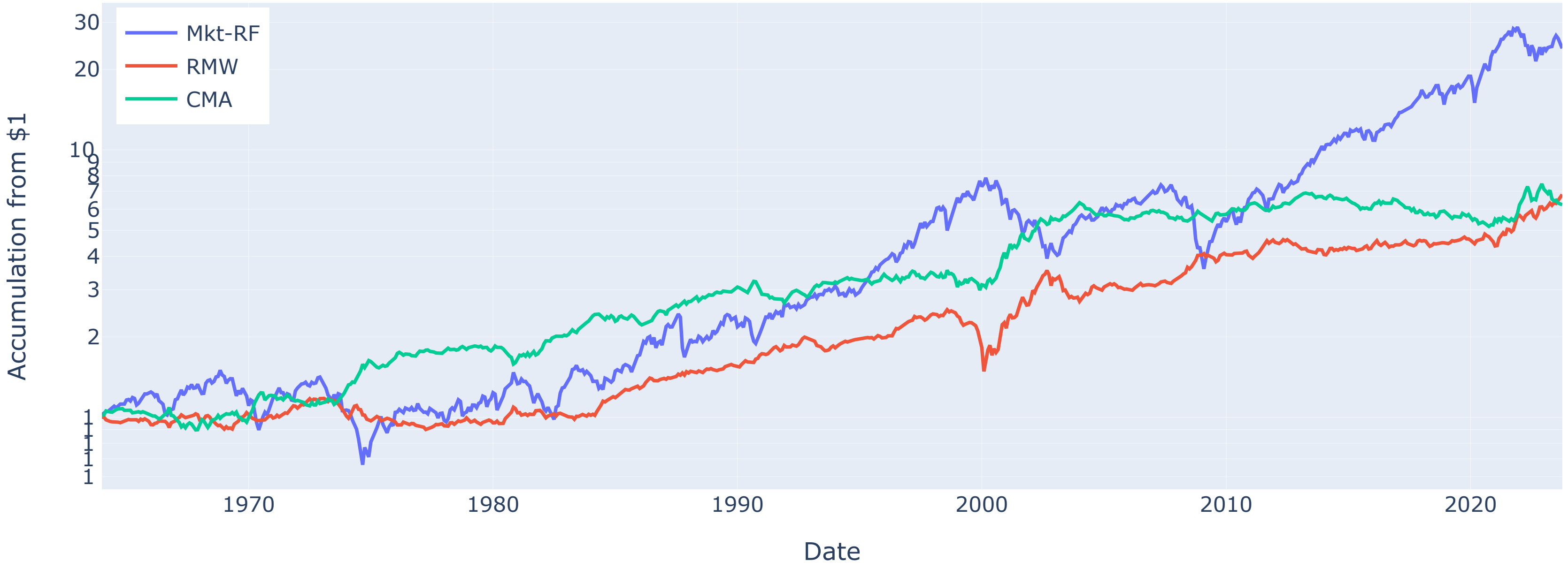
Fama-French 5-factor model

- Industrious researchers have continued to generate firm characteristics that correlate with ex post performance.
- Recently, Fama and French have argued for the following model:

$$
\begin{aligned}
R_{i,t} - R_{f,t} = \alpha_i + \beta_i (R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t \\
+ r_i RMW_t + c_i CMA_t + \varepsilon_{i,t}
\end{aligned}
$$

- Size factor: SMB (Small Minus Big)
- Value factor: HML (High Minus Low)
- Operating profitability factor: RMW (Robust Minus Weak)
- Investment factor: CMA (Conservative Minus Aggressive)

RMW + CMA cumulative returns



(Data starts in the 1960s due to availability of accounting information.)

What are the CAPM alphas of HML and SMB?

$$RMW_t = \alpha_{\text{RMW}} + \beta_{\text{RMW}}(R_{m,t} - R_{f,t}) + \varepsilon_{i,t}$$

$$CMA_t = \alpha_{\text{CMA}} + \beta_{\text{CMA}}(R_{m,t} - R_{f,t}) + \varepsilon_{i,t}$$

Expected return estimates

Factor models and E[r] estimates

- For a given stock, we need three ingredients to construct an expected return estimate.

1. Factor loadings $(\beta_i, s_i, h_i, r_i, c_i)$
2. Factor risk premia $(\lambda_{\text{mkt}}, \lambda_{\text{smb}}, \lambda_{\text{hml}}, \lambda_{\text{rmw}}, \lambda_{\text{cma}})$
3. The risk-free rate

- We have previously discussed the market risk premium.
- Now we want to estimate the other risk premiums
  - we can use the time-series average return of their respective long-short portfolio

E[r] estimates

Using the estimated factor loadings and estimates of the factor risk permia, the factor model's estimate of expected returns is:

$$E[R_i] = R_f + \hat{\beta}_i\hat{\lambda}_{\text{mkt}} + \hat{s}_i\hat{\lambda}_{\text{smb}} + \hat{h}_i\hat{\lambda}_{\text{hml}} + \hat{r}_i\hat{\lambda}_{\text{rmw}} + \hat{c}_i\hat{\lambda}_{\text{cma}}$$

- Let's look at an example on the dashboard
- A notebook implementing this approach is on Colab

Characteristic regressions

Fama-MacBeth cross-sectional approach

- We could also simply use the cross-sectional relationship between realized returns and lagged characteristics to characterize expected returns.
- We will use N characteristics guided by the empirical record so far

The procedure

1. Run cross-sectional regressions for 120 months of returns

$$R_{it} - R_{ft} = a_t + \sum_{j=1}^{N} b_{j,t} \cdot \text{characteristic}_{it-1} + e_{it}$$

2. Take average of each characteristic's time-series of b_{jt}s

$$\bar{b}_j = \frac{1}{120} \sum_{t=1}^{120} b_{j,t}$$

3. Expected return estimate τ is:

$$E[R_{i\tau}] = R_{f\tau} + \bar{a} + \sum_{j=1}^{N} \bar{b}_j \cdot \text{characteristic}_{i\tau-1}$$

- Let's look at how to implement this

For next time: Fixed Income: Duration