Fixed Income: Convexity

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BUSI 448: Investments

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Where are we?

Last time:

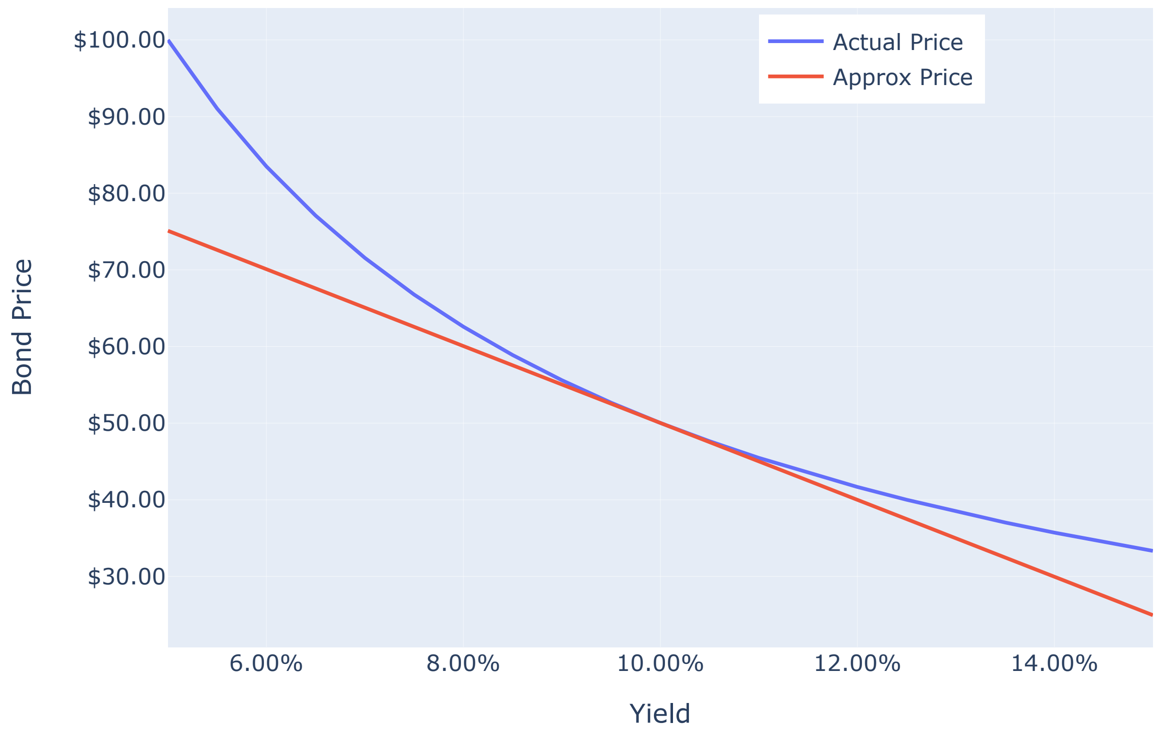
- Interest rate risk
- Duration

Today:

- More interest rate risk
- Convexity
- Callable bonds

Convexity

Interest rate risk and duration



- Duration allows a linear approximation of the price-yield relationship
  - Where and why is it a bad approximation?

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Moving beyond the linear approximation

- Can we improve on the relationship?
  - Hint: think back to your math classes

$$P(y + \Delta y) \approx P(y) + \frac{dP}{dy} \cdot \Delta y + 0.5 \cdot \frac{d^2 P}{dy^2} \cdot (\Delta y)^2.$$

Expressed in returns, rather than prices:

$$\frac{\Delta P}{P(y)} \approx \frac{1}{P} \cdot \frac{dP}{dy} \cdot \Delta y + 0.5 \frac{1}{P} \frac{d^2 P}{dy^2} \cdot (\Delta y)^2.$$

Convexity

- **Convexity** captures curvature of the pricing function
  - second derivative of price w.r.t. yield, scaled by price.

$$\text{convexity} = \frac{1}{P} \cdot \frac{d^2 P}{dy^2}$$

- For coupon bonds,

$$\text{convexity} = \frac{1}{(1 + y/m)^2} \left[\sum_{i=1}^{T} \frac{i(i+1)}{m^2} \cdot \frac{PV(CF_{t_i})}{P} \right].$$

Price and return approximations

The second-order price approximation is:

$$P(y + \Delta y) \approx P(y) - \text{mduration} \cdot P(y) \cdot \Delta y + 0.5 \cdot \text{convexity} \cdot P(y) \cdot (\Delta y)^2.$$

The second-order return approximation is:

$$\frac{\Delta P}{P(y)} \approx -\text{mduration} \cdot \Delta y + 0.5 \cdot \text{convexity} \cdot (\Delta y)^2.$$

- Let's take a look at today's notebook to see how this approximation performs.

Desirability of convexity

Positive convexity is desirable for investors

- For a fixed rate change magnitude, bond prices rise when rates fall by *more* than they fall when rates rise
- Example: coupon bonds

Negative convexity is undesirable for investors

- Instead, bond issuers like negative convexity
- Examples: callable bonds, mortgages

Callable Bonds

Call Schedules

Callable bond: the issuer has the right to call (repurchase) the bond at specified times at pre-determined price(s)

- usually a **call schedule** with call prices at specified call dates
  - first call price may be at premium over par value
  - call prices step down toward par later in call schedule
  - investors may be protected against call for an initial window
- issuers usually offer a higher coupon as compensation for the call option

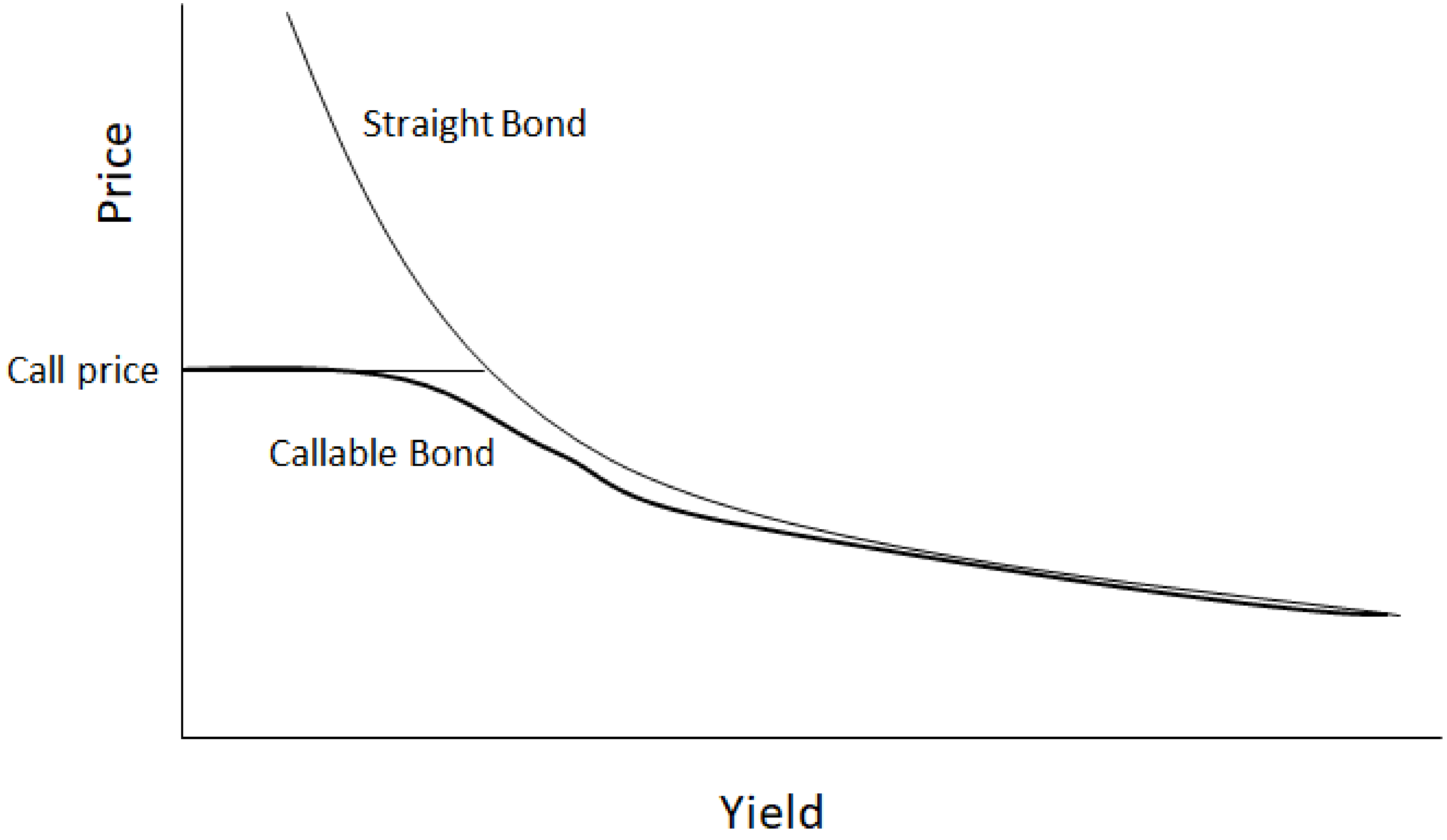
Interest rate risk

If rates fall,

- the bond price rises,
- the PV of future payment obligations for the firm may exceed the call price of the bond,
- the issuer benefits from calling the bond and reissuing debt at a lower coupon rate.

This creates a ceiling for the bond value at the call price.

Callable vs. straight bond prices



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Interest rate risk

At low interest rates, callable debt exhibits **negative convexity**.

- For a fixed rate change magnitude, bond prices rise when rates fall by *less* than they fall when rates rise
- this is undesirable for investors (hence higher coupon rates as compensation)

An aside: Yield to Call

We can calculate the IRR of paying today's price and receiving cash flows to a call date:

$$P = \sum_{t=1}^{T_{\text{call}}} \frac{C}{(1 + \frac{y_{\text{call}}}{m})^t} + \frac{\text{Call Price}}{(1 + \frac{y_{\text{call}}}{m})^{T_{\text{call}}}}$$

- y_{call}: the annual **yield-to-call**
- T_{call}: number of periods until the assumed call date
- m: number of payments per year

Estimating Duration and Convexity

Modified Duration

- Suppose we observe prices at three yields
  - $P_0 \equiv P(y_0)$
  - $P_+ \equiv P(y_0 + \Delta y)$
  - $P_- \equiv P(y_0 - \Delta y)$

An empirical estimate of modified duration at y_0 is:

$$\widehat{\text{mduration}} = \frac{1}{P_0} \frac{P_- - P_+}{2\Delta y}.$$

Convexity

An empirical estimate of convexity at y_0 is:

$$\widehat{\text{convexity}} = \frac{1}{P_0} \frac{(P_- - P_0) - (P_0 - P_+)}{(\Delta y)^2}.$$

For next time: Credit Risk